



Test Papers
Hub.

VCE Specialist Mathematics Practice Paper 1

Units 3 & 4 — Practice Examinations 1 & 2

Exam 1: 10 short-answer questions (40 marks) · Exam 2: 20 multiple-choice + 5 extended-response (80 marks) · 120 marks in all

How to use this booklet

- This paper contains BOTH examinations for a complete practice sitting. Examination 1 is technology-free (1 hour); Examination 2 allows an approved CAS calculator and one bound reference (2 hours).
- Exam 2 Section A has 20 multiple-choice questions (A–D); Section B has 5 extended-response questions. A formula sheet is assumed to be provided, as in the real examination.
- Show all working where more than one mark is available. Worked solutions, with a key takeaway for every question, follow the papers.

EXAMINATION 1 (TECHNOLOGY-FREE)

40 marks · 1 hour · No calculator or notes

Attempt Questions 1 – 10. Answer all questions in the spaces provided and show your working.

Question 1 (4 marks)

- (a) Let $z_1 = -2 + 3i$ and $z_2 = 1 + 3i$. Find $z_1 z_2$ and z_1/z_2 in the form $a + bi$. [2 marks]

- (b) Express $z = -1 + \sqrt{3}i$ in polar form $r \operatorname{cis}(\theta)$, where $-\pi < \theta \leq \pi$. [2 marks]

Question 2 (4 marks)

- (a) Solve $z^4 = 1$ for z in $\mathbb{C}$, giving the solutions in polar form $r \operatorname{cis}(\theta)$ with $-\pi < \theta \leq \pi$.

Question 3 (4 marks)

- (a) Given $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = -3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$, find $\mathbf{a} \cdot \mathbf{b}$. [1 mark]

- (b) Find the cosine of the angle between $\mathbf{a}$ and $\mathbf{b}$. [2 marks]

- (c) Find the scalar resolute of $\mathbf{a}$ in the direction of $\mathbf{b}$. [1 mark]

Question 4 (4 marks)

- (a) Find the vector cross product $\mathbf{a} \times \mathbf{b}$, where $\mathbf{a} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$. [2 marks]

- (b) Hence find the area of the triangle with sides $\mathbf{a}$ and $\mathbf{b}$. [2 marks]

Question 5 (4 marks)

- (a) Use a suitable substitution to find $\int \sin(x)^3 \cdot \cos(x) \, dx$. [3 marks]

- (b) Hence evaluate the definite integral from $x = 0$ to $x = \frac{\pi}{2}$. [1 mark]

Question 6 (4 marks)

- (a) Find $\int x \cdot \ln(x) \, dx$.

Question 7 (4 marks)

- (a) Solve the differential equation $\frac{dy}{dx} = 5y$ given that $y = 4$ when $x = 0$.

WORKED SOLUTIONS

Answers & worked solutions

Multiple-choice answer key

1	A	2	A	3	C	4	B	5	C	6	D
7	C	8	B	9	B	10	D	11	A	12	D
13	C	14	B	15	A	16	D	17	D	18	A
19	B	20	C								

EXAMINATION 1 (TECHNOLOGY-FREE)

Question 1 (4 marks) [Complex numbers]

(a) Answer: $z_1 z_2 = -11 - 3i$; $z_1/z_2 = \frac{7}{10} + 9/10i$

- $z_1 z_2 = (-2 + 3i)(1 + 3i) = -11 - 3i$.
- z_1/z_2 : multiply top and bottom by the conjugate $1 - 3i$: $(-2 + 3i)(1 - 3i)/((1 + 3i)(1 - 3i)) = \frac{7 + 9i}{10}$.
- $z_1/z_2 = \frac{7}{10} + 9/10i$.

(b) Answer: $z = 2 \operatorname{cis}(\frac{2\pi}{3})$

- $r = |z| = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$.
- Plot the point to find the quadrant; $\theta = \frac{2\pi}{3}$.

LOS: Specialist: complex numbers

KEY TAKEAWAY

Divide complex numbers by multiplying by the conjugate; polar form needs $r = |z|$ and the argument from the correct quadrant.

Question 2 (4 marks) [Roots of complex numbers]

(a) Answer: $z = 1 \operatorname{cis}(0), 1 \operatorname{cis}(\frac{\pi}{2}), 1 \operatorname{cis}(\pi), 1 \operatorname{cis}(\frac{3\pi}{2})$

- Write 1 in polar form: $1 = 1 \operatorname{cis}(0)$.
- Take the fourth root of the modulus: $r = 1$; divide the angle by 4 and add $2\pi j/4$ for $j = 0, \dots, 3$.
- Solutions: $1 \operatorname{cis}(0), 1 \operatorname{cis}(\frac{\pi}{2}), 1 \operatorname{cis}(\pi), 1 \operatorname{cis}(\frac{3\pi}{2})$.

LOS: Specialist: complex numbers

KEY TAKEAWAY

De Moivre for roots: take the n th root of the modulus, divide the argument by n , and add $\frac{2\pi}{n}$ for each further root.